## ON THE CIRCULATING BOILING BED ENERGY

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Energy losses through gas filtration in a circulating boiling bed have been analyzed. The expression of the power needed for sustaining the internal circulation of particles has been obtained. On this basis the theoretical dependence for determining the carrying capacity of the flow and the lower boundary of existence of a circulating boiling bed (minimum transport velocity) has been established. Simple engineering formulas for calculating the bed drag and the maximum transport velocity have been obtained.

**Keywords:** circulating boiling bed, carrying capacity of the flow, minimum and maximum transport velocity of gas, bed drag.

**Introduction.** The circulating boiling bed (CBB) is a complex pneumatic transport system in which two circulating loops — external and internal ones — exist simultaneously (Fig. 1). Particles continuously migrate from one loop to the other by the diffusion mechanism [1]. Moreover, in the bed there exists a directed (convective) flow of particles from the core into the annular (near-wall) region that provides a considerable decrease with height in the density of particles at practically constant values of their velocity (for more detail see [2]). It should be noted that the nature of convective flow of particles in the direction of the flow riser walls is not clearly understood. This phenomenon is probably associated with the determinancy of the gas profile in the riser (at the center the velocity is higher than near the wall) and the instability of the system "in the small" (according O. M. Todes' terminology [3, p. 64] (with respect to the zonal porosity perturbations in the transverse direction). This leads to the appearance of "limiting cycles" in the form of convective flows of particles into the annular region, indicating that the CBB is a system stable "in the large." As is known [3], in an ordinary boiling bed in which particle densities are much higher than in a CBB, instability "in the small" leads to "limiting cycles" in the form of macroscopic formations — gas bubbles passing through the bed with a fairly stable frequency. The noted pattern of the particle motion in the CBB increases the residence time of particles in the riser and creates favorable conditions for conducting in it various gas–solid processes.

The complexity of the CBB dynamics and the presence in the lower part of the bed of a concentrated twophase medium — bottom boiling bed — impede the calculation of plants with a CBB, in particular, their hydrodynamical drag. In [4], an attempt was made to generalize formally the experimental data on the drag of the CBB riser on the basis of the theory of similarity of transfer processes in disperse systems with suspended particles [5]. The following dependence has been obtained:

$$\frac{\Delta p_{\rm r}}{J_{\rm s} \left(u - u_{\rm t}\right)} = 0.6 {\rm Fr}_{\rm t}^{-1.68} \tag{1}$$

for  $Fr_t = 0.01 - 0.4$ .

In the present work, which is a sequel of [4], we seek to obtain, by analyzing the energy loss of the gas filtering through the CBB, a model dependence of the riser drag on the determining factors. Such a dependence, reflecting the contributions of different components  $\Delta p_r$ , will make it possible to establish the boundaries of existence of a CBB as a pneumatic system with developed internal circulation of particles.

The excess power of the gas (fan) with account for the above-mentioned pattern of the particle motion in the bed (Fig. 1) is expended in:

1) raising particles in the core (increasing their potential energy)

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Fig. 1. Scheme of the gas and particle flows in a circulating boiling bed.

$$N_1 = \int_{H_{fb}}^{H} A\rho_1 u_1 g dx ; \qquad (2)$$

2) increasing the momentum of particles arriving at the riser from the lowering part of the external circulation loop

$$N_2 = J_s \left(\frac{u}{A} - u_t\right)(u - u_t) \tag{3}$$

provided that the velocity of particles in the CBB core is constant and equal to  $\frac{u}{A} - u_t$  and the velocity of particles arriving at the riser from the lowering loop is equal to zero;

3) increasing the momentum of particles flowing into the bottom bed from the internal circulation loop with a velocity  $(-u_2)$ :

$$N_{3} = B\rho_{2}u_{2}\left(\frac{u}{A} - u_{t} + u_{2}\right)(u - u_{t}) \Big|_{x = H_{fb}};$$
(4)

4) filtering the gas through the bottom boiling bed

$$N_4 = g\rho_{\rm fb}H_{\rm fb}\left(u - u_{\rm t}\right);\tag{5}$$

5) friction of the gas and particles against the riser walls  $(N_5)$ . In [1], it has been shown that this component can be neglected, since the phase velocities in the near-wall region of the CBB are, as a rule, low compared to the bed core.

Thus, the total power expenditures are

$$N_{\Sigma} = N_1 + N_2 + N_3 + N_4 \,. \tag{6}$$

To calculate the  $N_i$  components, the following empirical dependences are used:

$$\frac{u_2}{(u-u_t)} = 0.1 \mathrm{Fr}_t^{-0.7} \,, \tag{7}$$

$$\frac{\rho}{\rho_{\rm s}} = 1 - \varepsilon = \overline{J}_{\rm s} \left( x' \right)^{-0.82},\tag{8}$$

$$1 - \varepsilon_{\rm fb} = 0.33 {\rm Fr}_{\rm t}^{-0.045}$$
, (9)

$$\frac{H_{\rm fb}}{H} = 1.25 {\rm Fr}_{\rm t}^{-0.8} \,\overline{J}_{\rm s}^{1.1} \,, \tag{10}$$

obtained in [4, 6] as a result of the generalization of numerous experimental data. Moreover, the following kinematic relations hold:

$$A\rho_1 + B\rho_2 = \rho , \quad A + B = 1 , \quad A\rho_1 u_1 - B\rho_2 u_2 = J_s , \quad \rho_2 = n\rho_1 , \tag{11}$$

where the coefficient n with account for (8) is equal to

$$n = \frac{A}{B} \frac{u_1' - (x')^{0.82}}{u_2' + (x')^{0.82}}.$$
(12)

Consider in more detail the  $N_i$  terms. For  $N_1$  in view of (11), assuming B = const, we have

$$N_{1} = g \int_{H_{fb}}^{H} (J_{s} + B\rho_{2}u_{2}) dx = g (H - H_{fb}) J_{s} + N_{in.c}, \qquad (13)$$

where  $N_{\text{in.c}} = Bg \int_{H_{\text{fb}}}^{H} \rho_2 u_2 dx$ . The first term in (13) is the power expended by the gas in increasing the potential energy

of particles of the external circulation loop, and the second one — in increasing the potential energy of particles of the internal circulation loop (power expended in internal circulation of particles). The last component of (13) is of particular importance for the analysis, since it is a distinguishing feature of the CBB. In view of (7), (8), (11), (12) and  $u'_1 \approx 1$ , it can be given in the form

$$N_{\rm in.c} = 0.1 \rho_{\rm s} \left(u - u_{\rm t}\right)^3 \frac{\overline{J}_{\rm s} {\rm Fr}_{\rm t}^{-1.7}}{1 + 0.1 {\rm Fr}_{\rm t}^{-0.7}} \left( 5.5 \left( 1 - \left(\frac{H_{\rm fb}}{H}\right)^{0.18} \right) - 1 + \frac{H_{\rm fb}}{H} \right).$$
(14)

For  $N_2$ , on the same assumption  $(u'_1 \approx 1)$  from (3) we obtain

$$N_2 \cong \overline{J}_{\rm s} \rho_{\rm s} \left( u - u_{\rm t} \right)^3. \tag{15}$$

Calculation of the component  $N_3$  by formula (4) is hindered because of the absence in the literature of recommendations for determining B,  $\rho_2$  and  $u_2$  in the annular region directly over the bottom boiling bed. Therefore, the  $N_3$  value was found from the equation

$$\frac{N_3}{N_2} = \frac{B\rho_2 u_2}{J_s},$$
(16)

which was obtained from the natural assumption that  $N_3/N_2$  is defined by the relation between the flows of internal and external circulation of particles. With the use of (7), (11) at  $n \approx 2$  and values of A = 0.85 and B = 0.15 [2] we have the following estimate of the  $N_3$  value:

$$N_3 \approx N_2 \frac{1}{30 \mathrm{Fr}_{\mathrm{t}}^{0.7} - 1},\tag{17}$$

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Fig. 2. Portion of the power expended in sustaining the internal circulation of particles: 1)  $\overline{J}_{s}^{*} = 10^{-5}$ ; 2)  $10^{-3}$ ; 3)  $10^{-2}$ .

which is applicable for  $Fr_t > 7.8 \cdot 10^{-3}$ .

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The  $N_4$  value in view of (9), (10) is equal to

$$N_4 \approx 0.42 g \rho_{\rm s} H \left( u - u_{\rm t} \right) \, {\rm Fr}_{\rm t}^{-0.8} \, \overline{J}_{\rm s}^{1.1} \,. \tag{18}$$

For the dimensionless excess power of the gas in view of (10)

$$N_{\Sigma}' = \frac{N_{\Sigma}}{\rho_{s} (u - u_{t})^{3}} = \overline{J}_{s} \left( 1 + \frac{1}{30 F r_{t}^{0.7} - 1} \right) + \frac{\overline{J}_{s}}{F r_{t}} \left( 1 - 1.25 F r_{t}^{-0.8} \overline{J}_{s}^{1.1} \right)$$
  
$$0.1 \frac{\overline{J}_{s} F r_{t}^{-1.7}}{1 + 0.1 F r_{t}^{-0.7}} \left( 5.5 \left( 1 - F r_{t}^{-0.14} \overline{J}_{s}^{0.2} \right) - 1 + 1.25 F r_{t}^{-0.8} \overline{J}_{s}^{1.1} \right) + 0.42 F r_{t}^{-1.8} \overline{J}_{s}^{1.1} .$$
(19)

Figure 2 shows, with account for  $\overline{J}_s = \overline{J}_s^* \sqrt{\mathrm{Fr}_t}$ , the calculation by (14) and (19) of the portion of excess power expended in sustaining in the bed the internal circulation of particles. The characteristic feature of the functions  $N_{\mathrm{in.c}}/N_{\Sigma} = f(\mathrm{Fr}_t, \overline{J}_s^*)$  is their extreme character. The position of the maximum and its value depend on  $\overline{J}_s^*$ . As is seen, the portion of internal circulation decreases with increasing  $J_s$  and increases with increasing height of the riser *H*. Another feature of  $N_{\mathrm{in.c}}/N_{\Sigma} = f(\mathrm{Fr}_t, \overline{J}_s^*)$  is the presence of a zero of this function determined by the equation

5.5 
$$\left(1 - \mathrm{Fr}_{\mathrm{t}}^{-0.14} \overline{J}_{\mathrm{s}}^{0.2}\right) - 1 + 1.25 \mathrm{Fr}_{\mathrm{t}}^{-0.8} \overline{J}_{\mathrm{s}}^{1.1} = 0$$
. (20)

Apparently, (20) is equivalent to the expression

$$5.5\left(1 - \left(\frac{H_{\rm fb}}{H}\right)^{0.18}\right) - 1 + \frac{H_{\rm fb}}{H} = 0, \qquad (21)$$

which has a unique solution  $H_{\text{fb}} = H$ . This means that internal circulation of particles disappears when the bottom bed fills the whole of the riser. From (13) it follows that, in so doing, external circulation also disappears and the CBB as a transport system changes over into a "stationary" boiling bed under turbulent conditions with a relatively small carryover of particles (despite the fact that  $u > u_t$ ). Solution (21) in view of (10) expresses the relation between Fr<sub>t</sub> and  $\overline{J}_s$ :

$$\frac{H_{\rm fb}}{H} = 1 = 1.25 {\rm Fr}_{\rm t}^{-0.8} \overline{J}_{\rm s}^{1.1} , \qquad (22)$$

which gives the relation

$$(Fr_t)_{\min} = \frac{(u_{\min} - u_t)^2}{gH} = 1.18 \left(\overline{J}_s^*\right)^{0.81},$$
(23)

determining the value of the minimum transport velocity  $u_{\min}$  capable of carrying the particle flow  $J_s$  (see Fig. 2). The inverse of (23)

$$(\bar{J}_{s}^{*})_{\text{lim}} = \frac{(J_{s})_{\text{lim}}}{\rho_{s}\sqrt{gH}} = 0.82 \text{Fr}_{t}^{1.23}$$
 (24)

expresses the limiting carrying capacity  $(J_s)_{\text{lim}}$  of the gas flow with velocity u. As is seen from (24),  $(J_s)_{\text{lim}}$  markedly increases with increasing excess speed of filtration:  $(J_s)_{\text{lim}} \sim (u - u_t)^{2.46}$ .

For the CBB drag from (19) we get

$$\frac{\Delta p_{\rm r}}{J_{\rm s} \left(u-u_{\rm t}\right)} = \frac{N_{\Sigma}^{\prime}}{\bar{J}_{\rm s}} = 1 + \frac{1}{30 {\rm Fr}_{\rm t}^{0.7} - 1} + \frac{\left(1 - 1.25 {\rm Fr}_{\rm t}^{-0.8} \bar{J}_{\rm s}^{1.1}\right)}{{\rm Fr}_{\rm t}}$$
$$+ \frac{0.1 {\rm Fr}_{\rm t}^{-1.7}}{1 + 0.1 {\rm Fr}_{\rm t}^{-0.7}} \left(5.5 \left(1 - {\rm Fr}_{\rm t}^{-0.14} \bar{J}_{\rm s}^{0.2}\right) - 1 + 1.25 {\rm Fr}_{\rm t}^{-0.8} \bar{J}_{\rm s}^{1.1}\right) + 0.42 {\rm Fr}_{\rm t}^{-1.8} \bar{J}_{\rm s}^{0.1}. \tag{25}$$

The calculation by Eq. (25) with account for  $\overline{J}_s = \overline{J}_s^* / \sqrt{Fr_t}$  and the experimental data of [1, 7–13] are given in Fig. 3. Here the empirical dependence (1) obtained in [4] has also been constructed. Analysis of (25) shows that at  $Fr_t < 0.03$  the bottom boiling bed offers the strongest resistance. Investigating (25) at large  $Fr_t$  is complicated by the absence of reliable recommendations for determining the transport velocity ( $u_{max}$ ) at which also internal circulation of particles disappears and the CBB switches to the regime of vertical pneumatic transport. This velocity can be determined on the basis of formula (25). To this end, let us simplify it somewhat. Taking into account the results of comparing the calculated and experimental values of  $\Delta p_r$  given in Fig. 3, as well as proceeding from the asymptotic behavior of function (25) at large and small  $Fr_t$ , we can use the following simple formula for calculating  $\Delta p_r$ :

$$\frac{\Delta p_{\rm r}}{J_{\rm s} \left(u - u_{\rm t}\right)} = 1 + 0.6 \,{\rm Fr_{\rm t}^{-1.68}} \,. \tag{26}$$

The results of the calculation by this formula are also shown in Fig. 3. Let us define the quantity  $u_{\text{max}}$  as a velocity at which the drag of the CBB and of the system of vertical pneumatic transport coincide. For the latter,  $\Delta p_{\text{r}}$  was taken in the form

$$\frac{\Delta p_{\rm r}}{J_{\rm s} \left(u - u_{\rm t}\right)} = 1 + {\rm Fr}_{\rm t}^{-1} + 0.68 \frac{H}{D} \left(\bar{J}_{\rm s}^{*}\right)^{-1/2} \frac{\rho_{\rm f}}{\rho_{\rm s}} \left(1 + {\rm Fr}_{\rm t}^{-1/2} \frac{u_{\rm t}}{\sqrt{gH}}\right)^2 {\rm Fr}_{\rm t}^{1/4} \,. \tag{27}$$

The first term on the right-hand side of (27) is the pressure differential due to the increase in the momentum of particles arriving at the riser; the second term is due to the increase in the potential energy of particles in the gravity field; and the third term is connected with the friction of the gas and particles against the riser wall  $(\Delta p_r)_{fr} = 2$ 

 $1.36 \frac{H}{D} \overline{J}_{s}^{1/2} \frac{\rho_{f} u^{2}}{2}$  [14], which is substantial in the case of pneumatic transport.

In view of (26) and (27) the  $u_{\text{max}}$  value is determined by the equation



Fig. 3. Drag of the circulating boiling bed and of the pneumatic transport system: 1–3) calculation by (25) at  $\overline{J}_{s}^{*} = 10^{-4}$ ,  $10^{-2}$ ,  $10^{-1}$ ; 4) by formula (1); 5) by formula (26); 6–8) calculation by (27) at  $\overline{J}_{s}^{*} = 0.06$ , 0.012, 0.018, 9) [1]; 10) [7]; 11) [8]; 12) [9]; 13) [10]; 14) [11]; 15) [12]; 16) [13].

$$\operatorname{Fr}_{t}^{-1} + 0.68 \frac{H}{D} \left( \overline{J}_{s}^{*} \right)^{-1/2} \frac{\rho_{f}}{\rho_{s}} \left( 1 + \operatorname{Fr}_{t}^{-1/2} \frac{u_{t}}{\sqrt{gH}} \right)^{2} \operatorname{Fr}_{t}^{1/4} - 0.6 \operatorname{Fr}_{t}^{-1.68} = 0 .$$
<sup>(28)</sup>

The numerical solution of (28) was approximated by the following relation:

$$(Fr_{t})_{max} = 0.47 - 0.68 \frac{H}{D} (\bar{J}_{s}^{*})^{-1/2} \frac{\rho_{f}}{\rho_{s}} \left( 0.12 + 0.2 \frac{u_{t}}{\sqrt{gH}} \right).$$
(29)

The method for determining  $u_{\text{max}}$  is illustrated in Fig. 3, which also shows the dependences  $\Delta p_r(J_s(u-u_t))$  for the operating conditions of [13] and their corresponding experimental data obtained in the regime of pneumatic transport.

**Conclusions.** The analysis of the expenditure of excess power of the gas (fan) in the CBB has made it possible to determine for the first time the value of the power expended in sustaining in the system the internal circulation of particles (14). On this basis we have obtained the dependence of the limiting carrying capacity of the flow on the determining factors (24) and established the lower boundary of existence of a CBB (23). It has been shown that there is good agreement between the theoretical dependence (25) for calculating the drag of the CBB riser and the experimental data. A simple engineering formula (26) for calculating  $\Delta p_r$  throughout the range of existence of a CBB has been obtained. Taking into account the laws describing the drag of the vertical pneumatic transport, we have established formula (29) for the maximum transport velocity of the gas which, jointly with (23), determines the range of existence of a CBB as a transport system with internal circulation of particles.

## NOTATION

A, portion of the cross-section of the bed occupied by rising particles (core of the bed); B, portion of the cross-section of the bed occupied by sinking particles (annular region); D, riser diameter, m;  $\text{Fr}_{t} = (u - u_{t})^{2}/gH$ , Froude number; g, gravitational acceleration, m/sec<sup>2</sup>; H, riser height, m;  $H_{\text{fb}}$ , height of the bottom boiling bed, m;  $H_{\text{fb}} = H_{\text{fb}}/H$ ;  $J_{\text{s}}$ , external circulation flow, kg/(m<sup>2</sup>·sec);  $\overline{J}_{\text{s}}^{*} = J_{\text{s}}/(\rho_{\text{s}}\sqrt{gH})$ ;  $\overline{J}_{\text{s}} = J_{\text{s}}/(\rho_{\text{s}}(u - u_{\text{t}}))$ ; N, specific power, kg/sec<sup>3</sup> (W/m<sup>2</sup>);  $\Delta p_{\text{r}}$ , pressure differential in the riser, N/m<sup>2</sup>;  $u_{\text{t}}$ , hovering velocity of a particle, m/sec; u, speed of filtration, m/sec;  $u_{1} = u/A - u_{\text{t}}$ ,  $u_{2}$ , particle velocity in the bed core and in the annular region, m/sec;  $u'_{1} = u_{1}/(u - u_{\text{t}})$ ;  $u'_{2} = u_{2}/(u - u_{\text{t}})$ ; x, vertical coordinate, m; x' = x/H;  $\varepsilon$ , porosity;  $\rho_{1}$ ,  $\rho_{2}$ , bed density in the core and in the annular region, kg/m<sup>3</sup>;  $\rho_{\text{f}}$ ,  $\rho_{\text{s}}$ , density of the gas and particles, kg/m<sup>3</sup>;  $\rho$ , mean in the horizontal section density

of the bed, kg/m<sup>3</sup>. Subscripts: f, gas; fb, bottom boiling bed; in.c, internal circulation; r, riser of the circulating boiling bed; s, particles; t, hovering conditions of a single particle;  $\Sigma$ , total; min, minimum; max, maximum; lim, limiting; fr, friction.

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